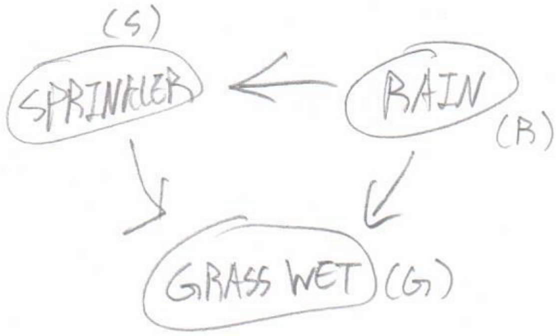


Conditional Probability Tables

(Sorry for confusing you at the practice !)

$P(S \mid B) = 0.01$

$P(\neg S \mid B) = 0.99$

$P(S \mid \neg B) = 0.4$

$P(\neg S \mid \neg B) = 0.6$

$P(B) = 0.2$

$P(\neg B) = 0.8$

$P(G \mid S, B) = 0.99$

$P(\neg G \mid S, B) = 0.01$

$P(G \mid S, \neg B) = 0.9$

$P(\neg G \mid S, \neg B) = 0.1$

$P(G \mid \neg S, B) = 0.8$

$P(\neg G \mid \neg S, B) = 0.2$

$P(G \mid \neg S, \neg B) = 0$

$P(\neg G \mid \neg S, \neg B) = 1$

You can calculate $P(G)$ like below, but utilising the concept of normalising constant seems to be the simpler way.

$$P(G) = P(G, S, B) + P(G, \neg S, B) + P(G, S, \neg B) + P(G, \neg S, \neg B)$$

$$= P(G \mid S, B) P(S \mid B) P(B) + P(G \mid \neg S, B) P(\neg S \mid B) P(B) + P(G \mid S, \neg B) P(S \mid \neg B) P(\neg B) + P(G \mid \neg S, \neg B) P(\neg S \mid \neg B) P(\neg B)$$

$$= 0.99 \times 0.01 \times 0.2 + 0.8 \times 0.99 \times 0.2 + 0.9 \times 0.4 \times 0.8 + 0 \times 0.6 \times 0.8$$

$$= 0.00198 + 0.1584 + 0.288 = 0.44838$$

Question : $P(B \mid G)$?

Solution :

$$P(B \mid G) \xrightarrow{\text{Bayes' rule}} = \frac{P(G \mid B) P(B)}{P(G)}$$

$$P(G \mid B) \xrightarrow{\text{marginal prob.}} = P(G, S \mid B) + P(G, \neg S \mid B)$$

conditional prob.

$$= P(G \mid S, B) P(S \mid B) + P(G \mid \neg S, B) P(\neg S \mid B)$$

$$= 0.99 \times 0.01 + 0.8 \times 0.99$$

$$= 0.0099 + 0.792 = 0.8019$$

$$P(B \mid G) = \frac{0.8019 \times 0.2}{P(G)} = \frac{0.16038}{P(G)}$$

$$P(\neg B \mid G) = \frac{P(G \mid \neg B) P(\neg B)}{P(G)},$$

$$P(G \mid \neg B) = P(G, S \mid \neg B) + P(G, \neg S \mid \neg B)$$

$$= P(G \mid S, \neg B) P(S \mid \neg B) + P(G \mid \neg S, \neg B) P(\neg S \mid \neg B)$$

$$= 0.9 \times 0.4 + 0 \times 0.6$$

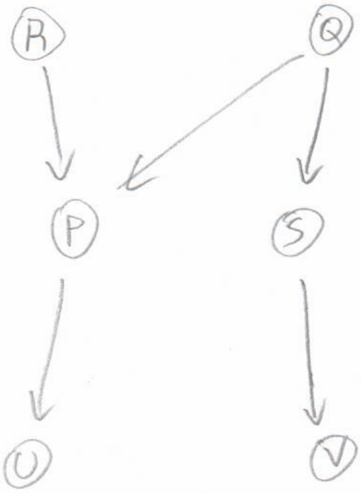
$$= 0.36 + 0 = 0.36$$

$$P(\neg B \mid G) = \frac{0.36 \times 0.8}{P(G)} = \frac{0.288}{P(G)}$$

$$P(B \mid G) + P(\neg B \mid G) = 1 = \frac{0.16038 + 0.288}{P(G)}$$

$$= \frac{0.44838}{P(G)}, \quad P(G) = 0.44838$$

$$P(B \mid G) = \frac{0.16038}{0.44838} \doteqdot 0.3577$$



← Polytree

$P(U|P) = 0.7$
$P(U|\neg P) = 0.2$

$P(B) = 0.01$ $\quad P(Q) = 0.05$

$P(P|B,Q) = 0.95$
$P(P|\neg B,Q) = 0.90$
$P(P|B,\neg Q) = 0.80$
$P(P|\neg B,\neg Q) = 0.01$

$P(S|Q) = 0.95$
$P(S|\neg Q) = 0.05$

$P(V|S) = 0.99$
$P(V|\neg S) = 0.1$

Q1) $P(U|B,Q,S)$ ↘ marginal prob.

$= P(U,P|B,Q,S) + P(U,\neg P|B,Q,S)$

↙ conditional prob.

$= P(U|P,B,Q,S)\,P(P|B,Q,S)$
$\quad + P(U|\neg P,B,Q,S)\,P(\neg P|B,Q,S)$

① Given the value for node Q, (tail-to-tail) undirected path between P and S is blocked. Hence P and S are conditionally independent. $(P \perp\!\!\!\perp S \mid Q)$

② Given the value for node P, (head-to-tail) undirected path between B and U and undirected path between Q and U are blocked. Hence $B \perp\!\!\!\perp U \mid P$.
$Q \perp\!\!\!\perp U \mid P$.

Hence $P(U|P,B,Q,S)\,P(P|B,Q,S)$
$\quad + P(U|\neg P,B,Q,S)\,P(\neg P|B,Q,S)$
$= P(U|P)\,P(P|B,Q) + P(U|\neg P)\,P(\neg P|B,Q)$

$= 0.7 \times 0.95 + 0.2 \times 0.05$
$= 0.665 + 0.01 = 0.675$

Q2) $P(P|Q)$

$= P(P,B|Q) + P(P,\neg B|Q)$
$= P(P|B,Q)\,P(B|Q)$
$\quad + P(P|\neg B,Q)\,P(\neg B|Q)$

① If the value for node P is not given, $B \perp\!\!\!\perp Q \mid P$.

Hence $P(P|B,Q)\,P(B|Q)$
$\quad + P(P|\neg B,Q)\,P(\neg B|Q)$

$= P(P|B,Q)\,P(B) + P(P|\neg B,Q)\,P(\neg B)$
$= 0.95 \times 0.01 + 0.9 \times 0.99$
$= 0.0095 + 0.891$
$= 0.9005$

Q3) $P(Q|P)$ ↘ Bayes' rule

$= \dfrac{P(P|Q)\,P(Q)}{P(P)} = \dfrac{0.9005 \times 0.05}{P(P)}$,

$P(\neg Q|P) = \dfrac{P(P|\neg Q)\,P(\neg Q)}{P(P)}$,

$P(P|\neg Q) = P(P,B|\neg Q) + P(P,\neg B|\neg Q)$
$\quad = P(P|B,\neg Q)\,P(B|\neg Q)$
$\quad + P(P|\neg B,\neg Q)\,P(\neg B|\neg Q)$

$B \perp\!\!\!\perp Q \mid P$, so $P(P|\neg Q) = P(P|B,\neg Q)P(B)$
$\quad + P(P|\neg B,\neg Q)P(\neg B)$

$= 0.8 \times 0.01 + 0.01 \times 0.99$
$= 0.008 + 0.0099 = 0.0179$

$P(\neg Q|P) + P(Q|P) = 1 = \frac{1}{P(P)}(0.0179 \times 0.95 + 0.045025)$

$P(P) = 0.017005 + 0.045025$
$\quad = 0.06203$

$P(Q|P) = \dfrac{0.045025}{0.06203}$
≈ 0.72586