Final Exam

Artificial Neural Networks & Computational Neuroscience
Seoul National University

Thursday, November 30, 2017

Answer the following 5 questions. Use no more space than one page for each question. Attempt to address the general theme of each problem using the subquestions as guidelines, rather than give fragmented answers to each subquestion. Feel free to specify your definitions and assumptions if you need them.

1. (**30 points**) Explain the following terms (i.e. definitions, properties, applications etc.) related to neurodynamic systems, neural networks, and machine learning.
   (a) Markov decision problem (b) Bellman's optimality equation
   (c) Lyapunov function (d) Content-addressable memory
   (e) Spurious attractors (f) Computational power of recurrent networks

2. (**30 points**) Consider temporal difference (TD) learning as a direct approximation method for dynamic programming. Derive the TD learning algorithm by following the (a)-(e) steps below.
   (a) Let the states be described by $\{i_n\}_{n=0}^{N}$. Let $g(i_n, i_{n+1})$ be the immediate cost for state transition from i_n to i_{n+1}. Let μ be the policy. Define the (recursive) cost-to-go function $J^{\mu}(i_n)$ of dynamic programming according to the Bellman equation.
   (b) Use the Robbins-Monro stochastic approximation to obtain the update rule for the cost-to-go function. Hint: The Robbins-Monro stochastic approximation is given as $r^{+} = (1-\eta)r + \eta g(r, \overline{v})$, where r is the old value, η is the step size, $\overline{r}$ is a random variable.
   (c) Define the temporal difference d_n in the update rule in (b). What is the meaning of the temporal difference? What does it compute?
   (d) Write the temporal difference learning rule, i.e. the update rule of $J(i_n)$ using d_n. This one-step update rule is called TD(0) algorithm.
   (e) Describe briefly how to generalize the TD(0) algorithm to TD(λ). Write the TD(λ) update rule. How is TD(1) different from TD(0)?

3. (20 points) Consider the BSB (brain-state-in-a-box) neurodynamic model.
   (a) Give the equations defining the BSB model in discrete-time form. Also

describe the neural network structure of the model. What is the activation function of the neurons? How are the neurons connected? Is this a recurrent neural network?

(b) Give the Lyapunove function of the BSB model. What do the equilibrium states of the BSB model represent? What might be a good application of this neural network? Explain or characterize the dynamics of the BSB model (you may use an example application to explain the properties).

4. (20 points) Consider the particle filter as a machine learning method for approximating the Bayesian filter in state-space models of dynamic systems.
   (a) Write the equations for the time update (state model or system model) and measurement update (observation model) for the Bayesian filter. Explain why this update procedure is a Bayesian inference process.
   (b) Particle filters (PFs) can be viewed as a sequential Monte Carlo approximation of Bayesian filtering. Explain why and how. Hints: Variations of PFs are also called a SIR (sampling-importance-resampling) or SIS (sequential importance sampling) algorithm. Explain all the ideas in PFs to make Bayesian filtering efficient, such as the relationship between Monte Carlo and sampling, the ideas of importance sampling, sequential Monte Carlo, and resampling.

5. (20 points) Consider the architectures and learning algorithms for recurrent neural network models.
   (a) Give (at least, three) different types of RNN network architectures. How does the connectivity structure of each of them look like? Give figures describing the network connectivity (architecture).
   (b) Describe the method of back-propagation through time (BPTT) to train the recurrent neural networks.
   (c) Describe the method of real-time recurrent learning (RTRL) to train the (fully-connected) recurrent neural networks.
   (d) How are the BPTT and RTRL different? What are the relative advantages and potential prices to pay for each of the two methods?

(Total 120 points)