Course 4190.101.001
Discrete Mathematics
Homework 1: Logic and Proofs

March 16, 2017

Due date: March 27, 2017, 23:59

Most of the questions here are selected from the textbook and these questions should only be used for your homework assignment. Your documents should be submitted as a **single** `pdf` file named "`hw1_STUDENT-ID.pdf`". For example, `hw1_2017-00000.pdf`. Email your `pdf` file to `snu.dm001@gmail.com` with title as "`HW1, STUDENT-ID, name`".

1. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

   a) $p \wedge q$ b) $p \to \neg q$ c) $\neg q \to p$

   d) $\neg p \to \neg q$ e) $p \leftrightarrow \neg q$ f) $\neg p \wedge (p \vee \neg q)$

2. Let p and q be the propositions
   p : It is below freezing.
   q : It is snowing.
   Write these propositions using p and q and logical connectives (including negations).

   a) It is below freezing but not snowing

   b) If it is below freezing, it is also snowing.

   c) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

   d) That it is below freezing is necessary and sufficient for it to be snowing.

3. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.

b) I go to the beach whenever it is a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

4. What is the value of x after each of these statements is encountered in a computer program, if $x = 1$ before the statement is reached?

   a) **if** $1 + 1 = 2$ **then** $x := x + 1$

   b) **if** $(2 + 3 = 5)$ AND $(3 + 4 = 7)$ **then** $x := x + 1$

   c) **if** $(1 + 1 = 2)$ XOR $(1 + 2 = 3)$ **then** $x := x + 1$

5. Each inhabitant of a remote village always tells the truth or always lies. A villager will only give a "Yes" or a "No" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit, the other branch leads deeper into the jungle. A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take?

6. Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said "Carlos did it." John said "I did not do it." Carlos said "Diana did it." Diana siad "Carlos lied when he said that I did it."

   a) If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.

   b) If the authorities also know that exactly one is lying, who did it? Explain your reasoning.

7. Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.

8. Show that $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology.

9. Show that $\{|\}$ (a set of logical operator which contains only $NAND$) is a complete collection of logcal operators.

10. Show that $p \mid (q \mid r)$ and $(p \mid q) \mid r$ not equivalent, so that the logical operator $\mid$ is not associative.

11. Let $P(x)$ be the statement "$x + 1 > 2x$." If the domain consisits of all integers, what are these truth values?

    a) $P(1)$

    b) $\exists x P(x)$

    c) $\forall x P(x)$

    d) $\exists x \neg P(x)$

12. Express the negation of these propositions using quantifiers, and then express the negation in English.

    a) Some driviers do not obey the speed limit.

    b) There is somoeone in this class who dows not have a good attitude.

13. Find a common domain for the variable x, y, z, and w for which the statement $\forall x \forall y \forall z \exists w((w \neq x) \wedge (w \neq y) \wedge (w \neq z))$ is true and another common domain for these variables for which it is false.

14. Translate these specifications into English where $F(p)$ is "Printer p is out of service," $B(p)$ is "Printer p is busy," $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued."

    a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

    b) $\forall p B(p) \rightarrow \exists j Q(j)$

    c) $\exists j(Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

    d) $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

15. Use quantifiers and logical connectives to express the fact that a quadratic polynomial with real number coefficients has at most two real roots.

16. Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true that $\forall x P(x) \vee \forall x Q(x))$ is true.

1. $\forall x(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal instantiation from (1)
3. $P(c)$	Simplification from (2)
4. $\forall x(P(x))$	Universal generalization from (3)
5. $Q(c)$	Simplification from (2)
6. $\forall x(Q(x))$	Universal generalization from (5)
7. $\forall x P(x) \vee \forall x Q(x))$	Conjunction from (4) and (6)

17. Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

18. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

    a) a proof by contraposition.

    b) a proof by contradiction.

19. Prove that $\sqrt[3]{2}$ is irrational.

20. Prove or disprove that you can use dominoes to tile a 5×5 checkerboard with three corners removed.

One domino = □■