Course 4190.101.001
Discrete Mathematics
Homework 3: Algorithms

April 11, 2017

Due: April 20, 2017, 23:59

Most of the questions here are selected from the textbook and these questions should only be used for your homework assignment. Your documents should be submitted as a **single** `pdf` file named "`hw3_STUDENT-ID.pdf`". For example, `hw3_2017-00000.pdf`. Email your `pdf` file to `snu.dm001@gmail.com` with title as "`HW3, STUDENT-ID, name`".

1. Determine whether each of these functions is $O(x)$.

   a) $f(x) = 10$ b) $f(x) = 3x + 7$ c) $f(x) = \lfloor x \rfloor$

2. Determine whether each of these functions is $O(x^2)$.

   a) $f(x) = x \log x$ b) $f(x) = x^4/2$ c) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

3. Describe an algorithm for finding the smallest integer in a finite sequence of natural numbers.

4. How many comparisons does the insertion sort use to sort the list $n, n-1, ..., 2, 1$?

5. Find functions f and g from the set of positive integers to the set of real numbers such that $f(n)$ is not $O(g(n))$ and $g(n)$ is not $O(f(n))$ simultaneously.

6. Show that $x^2 + 4x + 17$ is $O(x^3)$ but that x^3 is not $O(x^2 + 4x + 17)$.

7. Show that $(x^2 + xy + x \log y)^3$ is $O(x^6 y^3)$.

8. Give as good a big-O estimate as possible for each of these functions.

   a) $(n^2 + 8)(n + 1)$ b) $(n \log n + n^2)(n^3 + 2)$

   c) $(n! + 2^n)(n^3 + \log(n^2 + 1))$

9. What is the largest n for which one can solve in 10^6 seconds using an algorithm that requires $f(n)$ bit operations, where each bit operation is carried out in 1 second, with these values for $f(n)$.

   a) $\log n$ b) n c) $n \log n$

   d) n^2 e) 2^n f) $n!$

10. Describe the worst-case time complexity, measured in terms of comparisons, of the binary search algorithm.