# Course 4190.101.001
# Discrete Mathematics
# Homework 5: Counting and Discrete Probability

May 24, 2017

Due: June 1, 2017, 23:59

Most of the questions here are selected from the textbook and these questions should only be used for your homework assignment. Your documents should be submitted as a **single** `pdf` file named "`hw5_STUDENT-ID.pdf`". For example, `hw5_2017-00000.pdf`. Email your `pdf` file to `snu.dm001@gmail.com` with title as "`HW5, STUDENT-ID, name`".

1. How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s?

2. Let $n_1, n_2, ..., n_t$ be positive integers. Show that if $n_1 + n_2 + ... + n_t - t + 1$ objects are placed into $t$ boxes, then for some $i$, $i = 1, 2, ..., t$ the $i$th box contains at least $n_i$ objects.

3. Suppose that a department contains 10 men and 15 women. How many ways are there to from a committee with six members if it must have more women than men?

4. Prove the identity $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$, whenever $n$, $r$, and $k$ are nonnegative integers with $r \leq n$ and $k \leq r$,

   a) using a combinatorial argument.

   b) using an argment based on the formula for the number of $r$-combinations of a set with $n$ elements.

5. In this exercise we will count the number of paths in the $xy$ palne between the origin $(0,0)$ and the point $(m,n)$ such that each path is made up of a series of steps, where each step is a move one unit to the right or a move one unit upward. (No moves to the left or downard are allowed.)

a) Show that each path of the type described can be represented by a bit string consisting of $m$ 0s and $n$ 1s, where a 0 represents a move one unit to the right and a 1 represetns a move one unit upward.

b) Conclude from part (a) that there are $\binom{m+n}{m}$ paths of the desired type.

6. How many ways are there to pack nine identical DVDs into three indistinguishable boxes so that each box contains at least two DVDs?

7. How many terms are there in the expansion of $(x + y + z)^{100}$?

8. What is the probability that five-card poker hand contains two pairs (that is, two of each of two differnt kinds and a fifth card of a third kind)?

9. What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)

10. An electronics company is planning to introduce a new camera phone. The company commissions a marketing report for each new product that predicts either the success or the failure of the product. Of new products introduces by the company, 60% have been successes. Furthermore, 70% of their successful products were predicted to be successes, while 40% of failed products were predicted to be successes. Find the probability this new camera phone will be successful if its success has been predicted.

11. Ramesh can get to work three different ways: by bicycle, by car, or by bus. Because of commuter traffic, there is 50% change that he will be late when he drives his car. When he takes the bus, which uses a special lane reserved for buses, there is a 20% chance that he will be late. The probability that he is late when he rides his bicycle is only 5%. Ramesh arrives late one day. His boss wants to estimate the probability that he drove his car to work that day.

a) Suppose the boss assumes that there is a 1/3 chance that Ramesh takes each of the three ways he can get to work. What estimate for the probability that Ramesh drove his car does the boss obtain from Bayes' Theorem under this assumption?

b) Suppose the boss knows that Ramesh drives 30% of the time, takes the bus only 10% of the time, and takes his bicycle 60% of the time. What estimate for the probability that Ramesh drove his car does the boss obtrain from Bayes' Theorem using this informaton?

12. Show that $V(X + Y) = V(X) + V(Y) + 2\text{Cov}(X + Y)$.

13. When $m$ balls are distributed into $n$ bins uniformly at random, what is the probability that the first bin remains empty?