Acquisition of Children’s Addition Strategies

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2 + 3 = ?

- interviewed eight children weekly for 11 weeks
- each time ask them to solve 15 orally presented addition problems
- at first they know only the Sum strategy
- at the end seven of the eight children began to use the shortcut Min strategy
What are you seeing?
Problem

- Sum to Shortcut-Min Transition
- General Inductive Problem Solver
- Strategy Acquisition in the Addition Domain
- Conclusion
Two Intermediate Strategies

- SUM Strategy
- Shortcut Sum Strategy
- Min Strategy
- Shortcut MIN Strategy
Problem

Impasse-Driven?
Sum to Shortcut-Min Transition

- Problem
- Sum to Shortcut-Min Transition
- General Inductive Problem Solver
- Strategy Acquisition in the Addition Domain
- Conclusion
Comparison (1/2)

<table>
<thead>
<tr>
<th>Initial Sum</th>
<th>Shortcut-Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>left hand := first addend</td>
<td>left hand := larger addend</td>
</tr>
<tr>
<td>right hand := second addend</td>
<td>right hand := smaller addend</td>
</tr>
<tr>
<td>Counter := 0</td>
<td>Counter = left hand addend</td>
</tr>
<tr>
<td>Loop</td>
<td>Loop</td>
</tr>
<tr>
<td>Raise finger on left hand</td>
<td>Raise finger on right hand</td>
</tr>
<tr>
<td>Counter := Counter + 1</td>
<td>Counter := Counter + 1</td>
</tr>
<tr>
<td>Until Counter = left hand addend</td>
<td>Until #raised fingers =</td>
</tr>
<tr>
<td>Counter := 0</td>
<td>right hand addend</td>
</tr>
<tr>
<td>Loop</td>
<td></td>
</tr>
<tr>
<td>Raise finger on right hand</td>
<td>Raise finger on right hand</td>
</tr>
<tr>
<td>Counter := Counter + 1</td>
<td>Counter := Counter + 1</td>
</tr>
<tr>
<td>Until Counter = right hand addend</td>
<td>Until #raised fingers =</td>
</tr>
<tr>
<td></td>
<td>right hand addend</td>
</tr>
</tbody>
</table>
Comparison(2/2)

Counter := 0
Loop
  Mark raised finger
  Counter := Counter + 1
Until #marked fingers = #raised fingers
Let Answer be Counter

Let Answer be Counter
Problem

Sum to Shortcut-Min Transition

General Inductive Problem Solver

Strategy Acquisition in the Addition Domain

Conclusion
GIPS

- uses probabilistic induction to learn new preconditions on its operators
- thus can modify the descriptions of the operators themselves
A GIPS operator

COUNT(?Hand, ?Initvalue, ?Finalvalue)

**Preconditions:**
- Hand(?Hand)
- Just-raised(?Hand)
- Counter-value(?Initvalue)

**Add conditions:**
- Counter-value(?Finalvalue)

**Delete conditions:**
- Counter-value(?Initvalue)
- Just-raised(?Hand)

**Constraints:**
- ?Finalvalue is ?Initvalue + 1
GIPS Operators

**SELECT-HAND** Select an addend to be counted on each hand

**COUNT-OUT-LEFTHAND** Represent or count the left-hand addend

**START-COUNT** Keep track of the counter value while raising fingers

**COUNT** Count the last raised finger by incrementing the counter value

**LEFT-ADDEND-REPRESENTED** Stop counting and raising fingers on the left hand

**MARK-FINGER** Mark a finger that has already been raised
Strategy Acquisition in the Addition Domain

- Problem
- Sum to Shortcut-Min Transition
- General Inductive Problem Solver
- Strategy Acquisition in the Addition Domain
- Conclusion
Sum Strategy

Preconditions for LEFT-ADDEND-REPRESENTED

Sum Strategy (a):
  Raising(LeftHand)
  Assigned(LeftHand, ?Value)
  Counter-value(?Value)

Sum Strategy (b):
  Raising(LeftHand)
  Assigned(LeftHand, ?Value)
  Counter-value(?Value)
  Raised-fingers(LeftHand, ?Value)
Shortcut Sum Strategy

Preconditions for LEFT-ADDEND-REPRESENTED

Sum Strategy(b):
  Raising(LeftHand)
  Assigned(LeftHand, ?Value)
  Counter-value(?Value)
  Raised-fingers(LeftHand,?Value)

Shortcut Sum Strategy:
  Raising(LeftHand)
  Assigned(LeftHand, ?Value)
  Raised-fingers(LeftHand,?Value)
Min Strategy

Start counting with the larger addend on the left hand (at first!)
- Problem solver’s counter value is not equal to the number of fingers being raised on the right hand
- Children may fail to make correct answer
Shortcut Min Strategy

Preconditions for LEFT-ADDEND-REPRESENTED

Shortcut Sum Strategy:
Raising(LeftHand)
Assigned(LeftHand, ?Value)
Raised-fingers(LeftHand, ?Value)

Shortcut Min Strategy(b):
Raising(LeftHand)
Assigned(LeftHand, ?Value)
Conclusion

- Problem
- Sum to Shortcut-Min Transition
- General Inductive Problem Solver
- Strategy Acquisition in the Addition Domain
- Conclusion
The transition involves independent modifications to the Sum strategy

- parallelize
- making correct answer
- symbolic representation
References

- Randolph M. Jones, Kurt Vanlehn
  Acquisition of Children’s Addition Strategies: A Model of Impasse-Free, Knowledge-Level Learning

- Siegler R.S., Jenkins E.
  How children discover new strategies