Fall 2010 Graduate Course on
Dynamic Learning

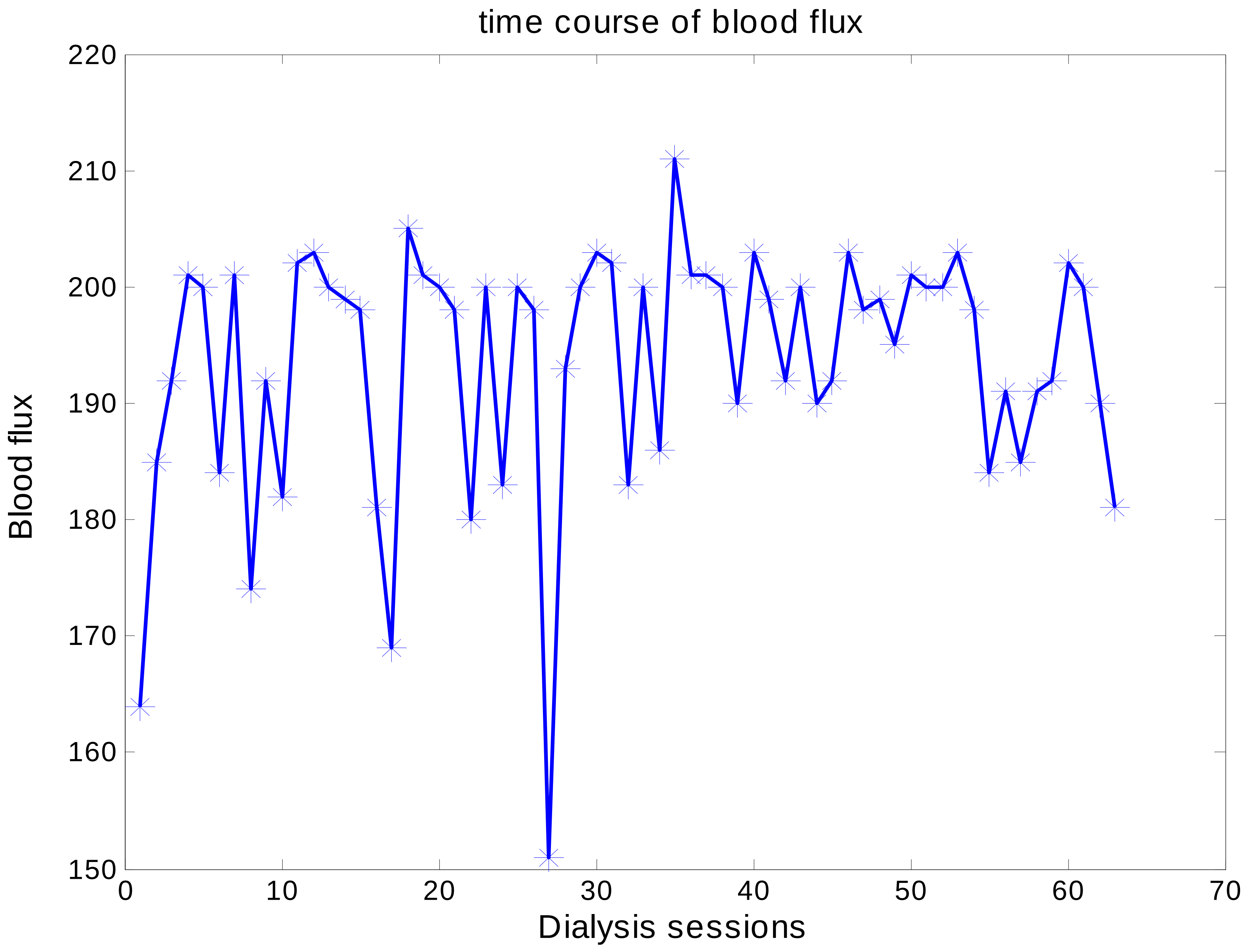
Chapter 2: Autoregressive Models

Byoung-Tak Zhang
http://bi.snu.ac.kr/~btzhang/

School of Computer Science and Engineering &
Cognitive Science and Brain Science Programs
Seoul National University

References

- Slides of Riccardo Bellazzi



time course of blood flux
Blood flux
Dialysis sessions
220
210
200
190
180
170
160
150
0
10
20
30
40
50
60
70

Time series

- Time series: a collection of observations made sequentially in time
- Many application fields:
  - Economic time series
  - Physical time series
  - Marketing time series
  - Process control
- Characteristics:
  - successive observations are NOT independent
  - The order of observation is crucial

Why time series analysis

- Description
- Explanation
- Prediction
- Control

Understand, then act

Modeling time series

- Time series: data are correlated; data are realiz ations of stochastic processes
- Stochastic linear discrete input-output models
- Two approaches:
  - Model the data as a function of time (a regression o ver time)
  - Model the data as a function of its past values: AR MA models
- Often, assumption of stationarity (the mean an d variance of the process generating the data do not change over time)

Autoregressive (AR) models

$$y_{k+1} = a_1 y_k + e_k$$

- AR(h) is a regression model that regresses eac h point on the previous h time points. Exampl e is AR(1)
- Each value is affected by random noise e_k with zero mean and variance σ^2
- Can be learned with linear estimation algorith m

Moving average (MA)

$$y_k = b_1 e_{k-1} + e_k$$

- A different kind of model is the Moving Avera ge model (MA(h))
- It propagates over time the effect of the rand om fluctuations
- The autocorrelation function may help in choo sing proper models
- An iterative estimation process is needed

ARMA

$$y_k = a_1 y_{k-1} + b_1 e_{k-1} + e_k$$

- It can be used to obtain a more parsimonious model, with “difficult” autocorrelation functions

ARMAX

- The system can be driven not only by no ise but also by eXogenous (control) input s

$$y_k = a_1 y_{k-1} + b_1 e_{k-1} + c_1 u_k + e_k$$

This is the general ARMAX model

Nonlinear models

- Also non-linear stochastic models have b een proposed in the literature
- Examples are NARX models

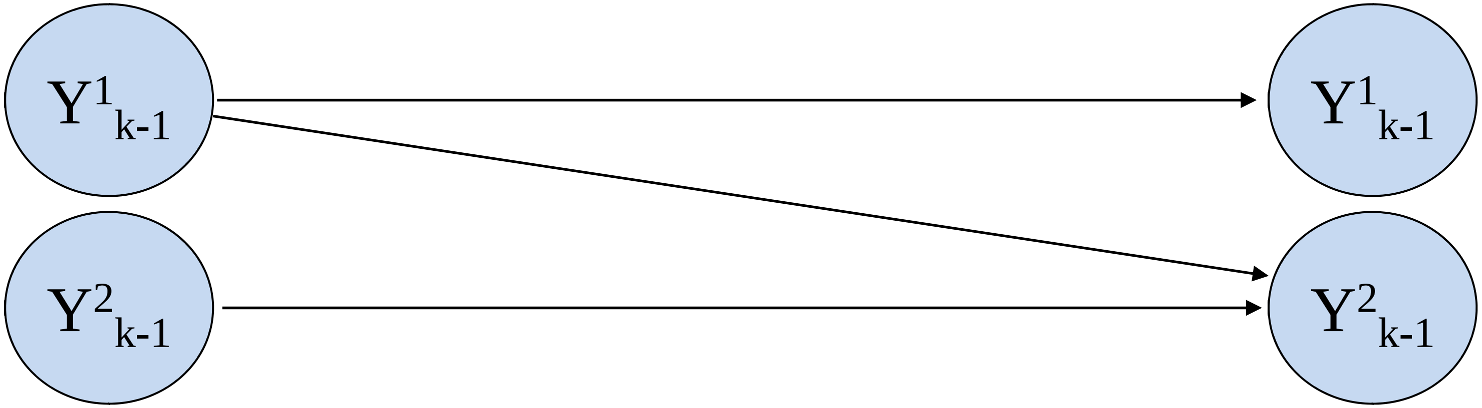
$$y_k = \sum d_k \varphi_k(y_{k-1}) + e_k$$

- NARX models can be easily learned from data with neural nets

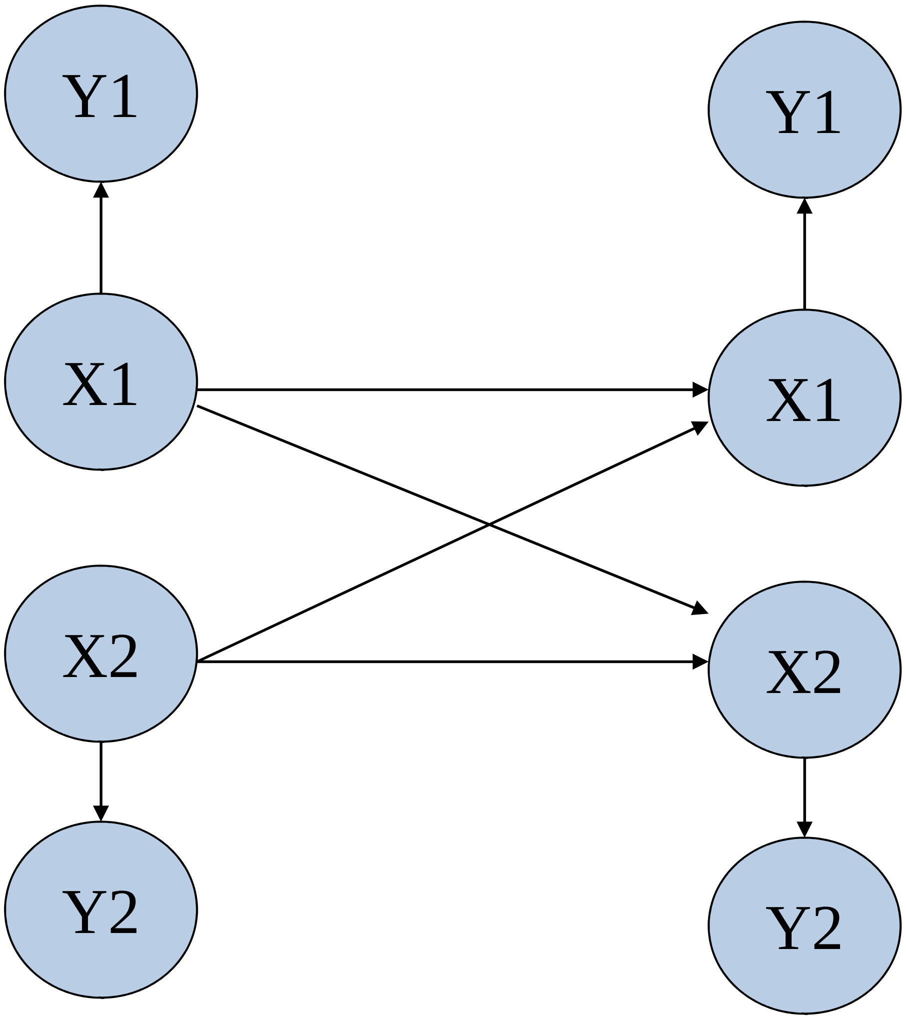
Structural Models

Nonlinear AR models

- Dynamic Bayesian nets

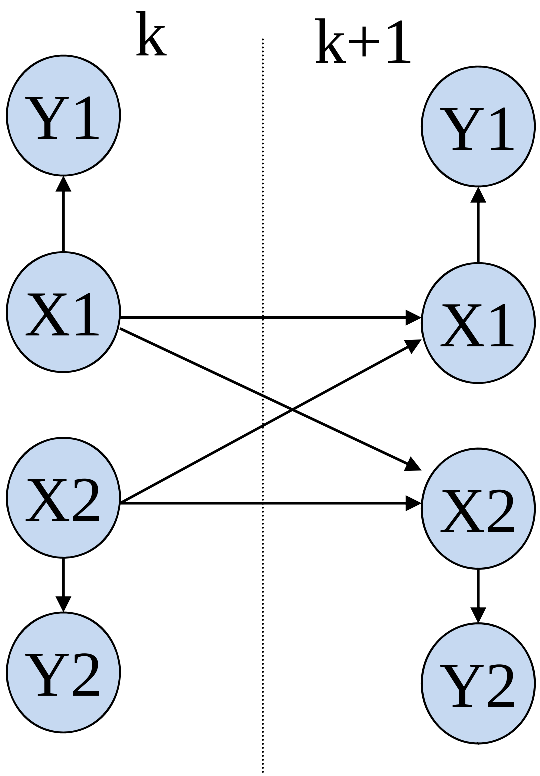


From black-box to structural stochastic models

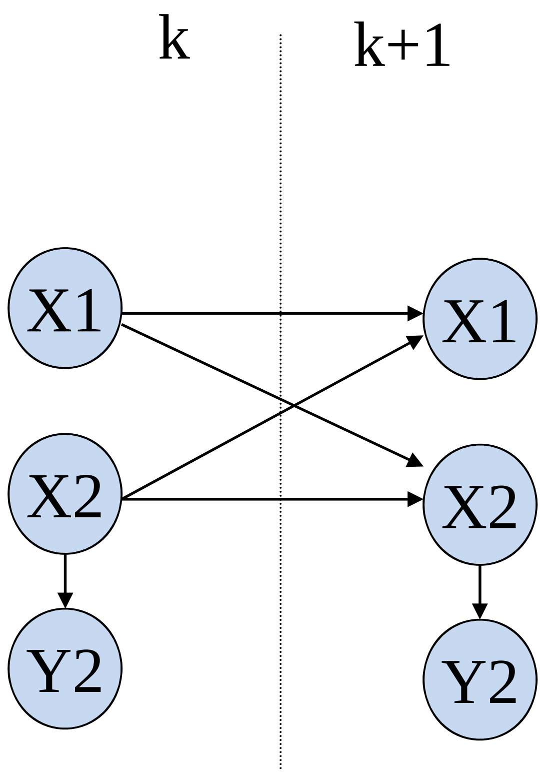


Examples:
- Kalman filters
- Dynamic BNs
- Hidden Markov Models

Observable and partially observable models



Fully observable



Partially observable