Predictive Coding: A Free-Energy Formulation

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Quiz

- What is the most important characteristic of the cortical system in the brain?. Please give an answer with the consideration of the structure of multilayered neural network and 6 layers in cortex.

Introduction

- Brain uses empirical Bayes for inference about its sensory input, given the hierarchical organization of cortical systems.

Hierarchical Dynamic Models (1/4)

- Hierarchical dynamic models are probabilistic generative models

$$p(y, \vartheta) = p(y \mid \vartheta)\, p(\vartheta),$$

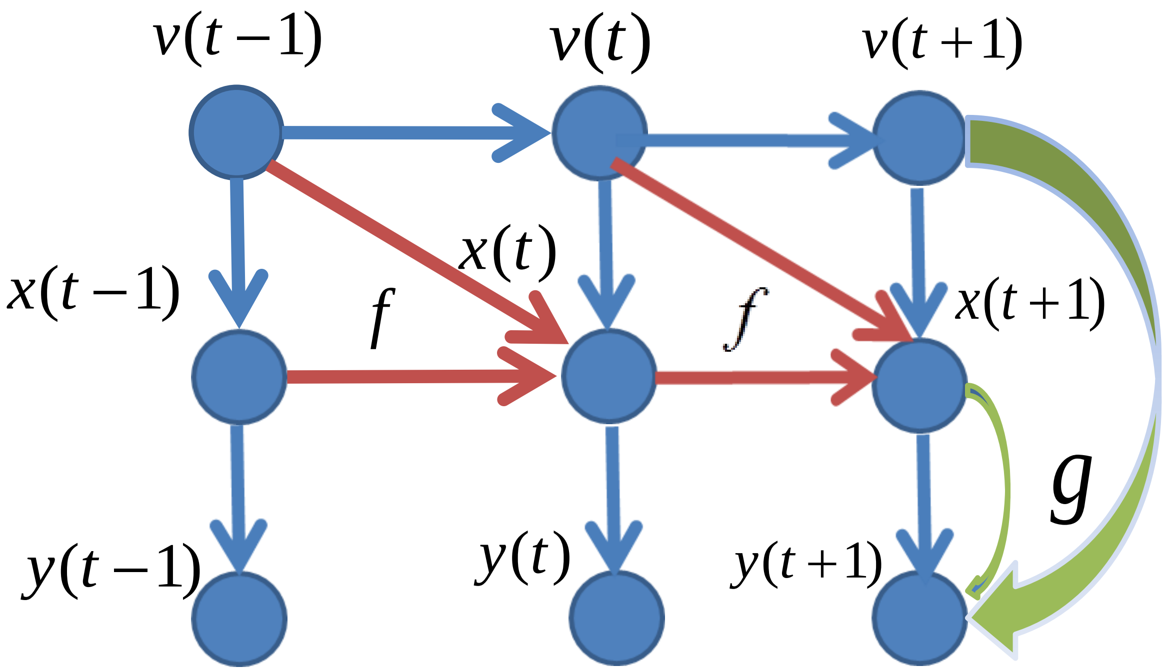
where y is the sensory data and $\vartheta = \{x, v, \Theta\}$ are parameters.

- v : inputs, sources, causes, or hidden states.
- x: mediate the influence of the input on the output and endow the system with memory.

Hierarchical Dynamic Models (2/4)

$$y = g(x, v) + z$$

$$\dot{x} = f(x, v) + w$$



Hierarchical Dynamic Models (3/4)

- $\tilde{y} = [y, y', y'', \ldots]^T$: generalized response comprising the position, velocity, acceleration jerk, and so on.

$$y = g(x,v) + z \qquad \dot{x} = x' = f(x,v) + w$$

$$y' = g_x x' + g_v v' + z' \qquad \dot{x}' = x'' = f_x x' + f_v v' + w'$$

$$y'' = g_x x'' + g_v v'' + z'' \qquad \dot{x}'' = x''' = f_x x'' + f_v v'' + w''$$

↓ ↓

$$\tilde{y} = \tilde{g} + \tilde{z} \qquad D\tilde{x} = \tilde{f} + \tilde{w}$$

Hierarchical Dynamic Models (4/4)

- $$\begin{aligned} p(\tilde{y},\tilde{x},\tilde{v}) &= p(\tilde{y}\,|\,\tilde{x},\tilde{v})\,p(\tilde{x},\tilde{v}) \\ p(\tilde{y}\,|\,\tilde{x},\tilde{v}) &= N(\tilde{y}:\tilde{g},\Sigma^{v}) \\ p(\tilde{x},\tilde{v}) &= p(\tilde{x}\,|\,\tilde{v})\,p(\tilde{v}) \\ p(\tilde{x}\,|\,\tilde{v}) &= N(D\tilde{x}:\tilde{f},\Sigma^{x}) \end{aligned}$$
- Plausible assumptions about noise are exploited by the brain.
- $\tilde{\Sigma}^{v},\tilde{\Sigma}^{x}$: covariances
- $\tilde{\Pi}^{v}, \tilde{\Pi}^{x}$: precisions which are the inverse of the covariances.

Model Inversion and Variational Bayes (1/5)

- Variational Bayes, which is a generic approach to model inversion that approximates the conditional density $p(\vartheta \mid \tilde{y}, m)$ on some model parameters, ϑ, given model m and data $\tilde{y}$.
- The log-evidence can be expressed in terms of a free-energy and divergence term

$$\ln p(\tilde{y} \mid m) = F + K(q(\vartheta) \parallel p(\vartheta \mid \tilde{y}, m)) \Rightarrow F = < \ln p(\tilde{y}, \vartheta) >_q - < \ln q(\vartheta) >_q$$

Model Inversion and Variational Bayes (2/5)

- The objective is to optimize the sufficient statistics of $q(\vartheta)$ by maximising the free-energy and minimizing the divergence.
- In physics, the free-energy is to be minimized and the free-energy definition in this paper is the negative of the conventional free-energy.

Model Inversion and Variational Bayes (3/5)

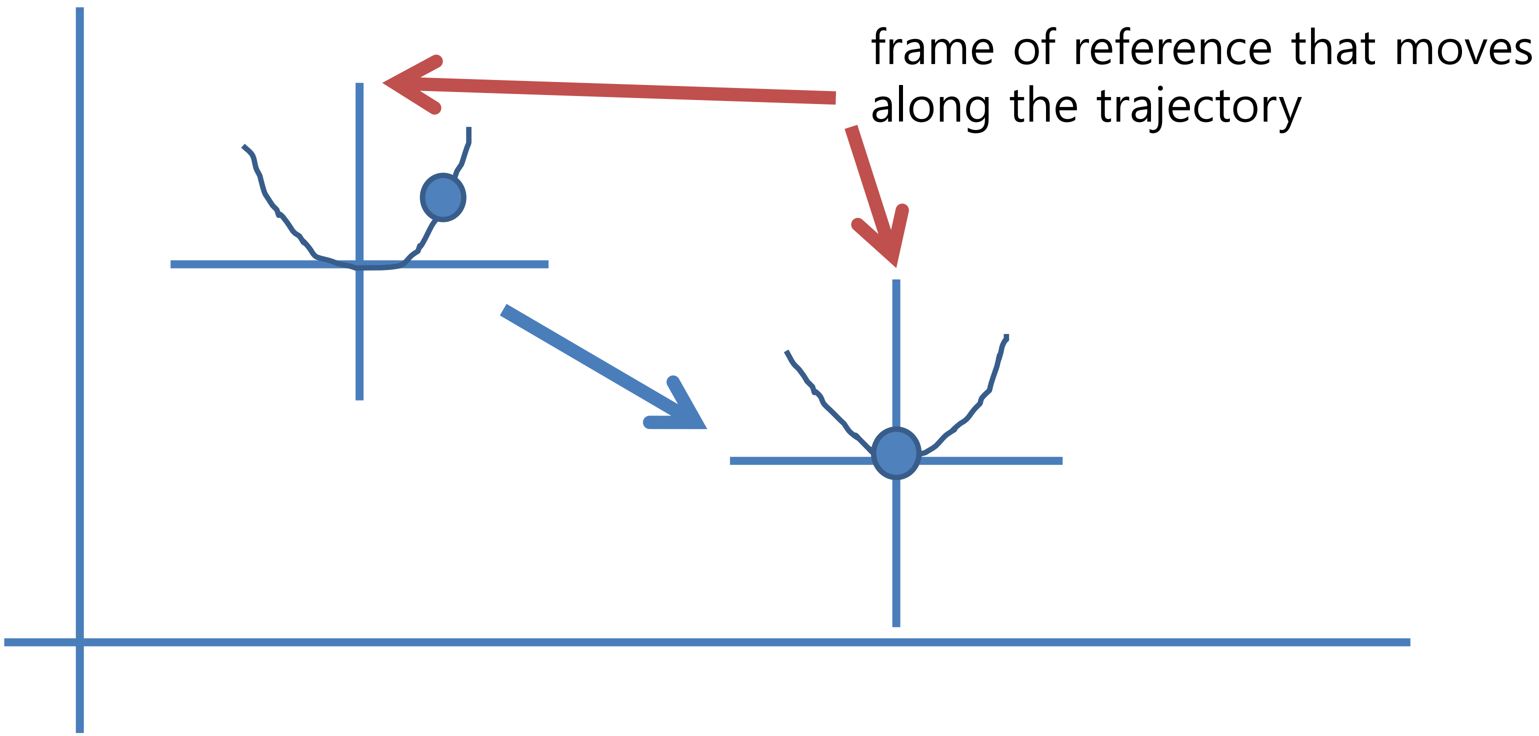
- We now seek $q(u)$ that maximizes the free energy at each point of time. This is proportional to exp(V), where V(t) is called the variational energy; this is the Gibbs energy expected under the conditional density of the parameters.
- $V(t) = U(t) = \ln p(\tilde{y}, u)$, where $u(t) = [\tilde{x}, \tilde{v}]^T$.
- Under the Laplace approximation, the conditional density assumes a fixed Gaussian form $q(u) = N(u : \tilde{\mu}, C)$.

Model Inversion and Variational Bayes (4/5)

- The conditional precision is a function of the mean. Therefore, we can reduce model inversion to optimizing one sufficient statistics; namely, the conditional mean. This is the solution to
  $$\dot{\tilde{\mu}} - D\tilde{\mu} = \partial_u V.$$
- $\dot{\tilde{\mu}} - D\tilde{\mu}$ can be regarded as motion in a frame of reference that moves along the trajectory encoded in generalized coordinates.

Model Inversion and Variational Bayes (5/5)

- If the gradient of the variational energy is zero, the mean of the motion becomes the motion of the mean, $\dot{\tilde{\mu}} = D\mu$



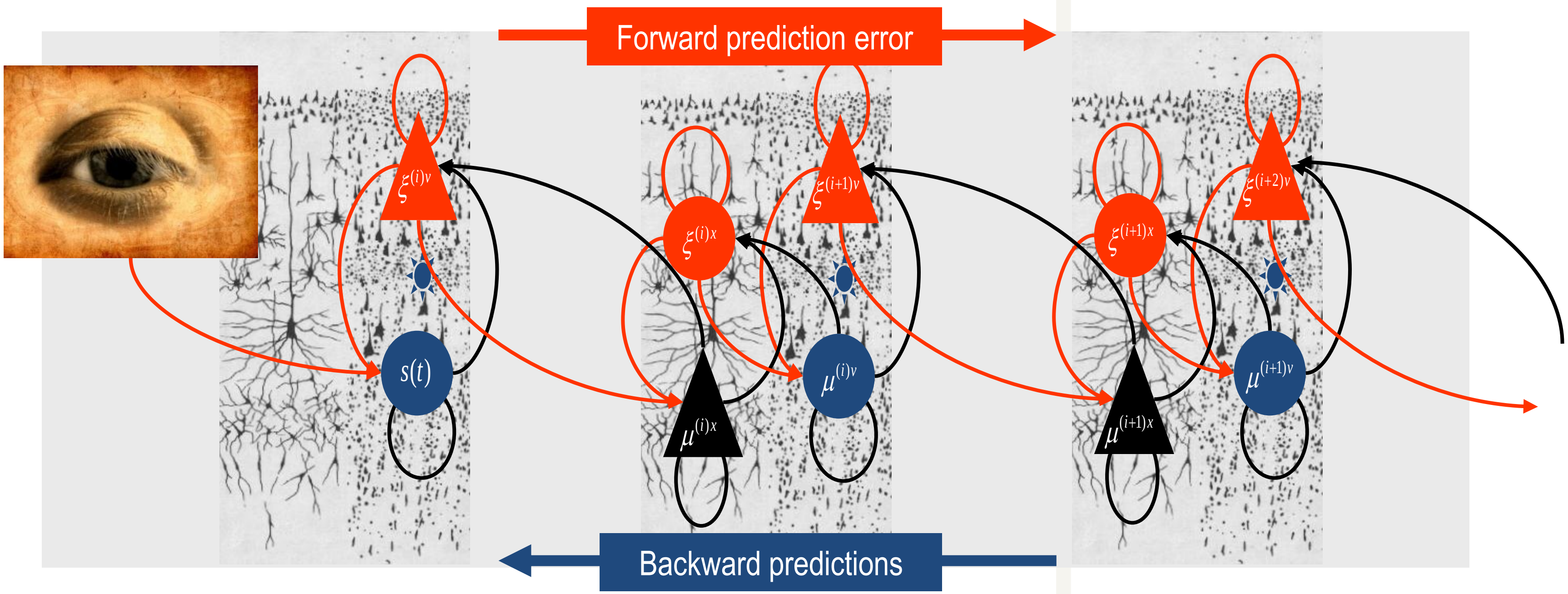
Hierarchical Models in the Brain

- A key architectural principle of the brain is the hierarchical organization.
- Forward connections arise largely in superficial pyramidal cells, in supra-granular layers, and terminate on spiny stellate cells of layer four in higher cortical areas.
- Backward connections arise largely from deep pyramidal cells in infra-granular layers and target cells in the infra and supra-granular layers of lower cortical areas.

Perception and message-passing

$$\xi^{(i)v} = \Pi^{(i)v}\varepsilon^{(i)v} = \Pi^{(i)v}(\mu^{(i-1)v} - g(\mu^{(i)}))$$

$$\xi^{(i)x} = \Pi^{(i)x}\varepsilon^{(i)x} = \Pi^{(i)x}(D\mu^{(i)x} - f(\mu^{(i)}))$$



$$\dot{\mu}^{(i)v} = D\mu^{(i)v} - \partial_v \varepsilon^{(i)T}\xi^{(i)} - \xi^{(i+1)v}$$

$$\dot{\mu}^{(i)x} = D\mu^{(i)x} - \partial_x \varepsilon^{(i)T}\xi^{(i)}$$

Synaptic plasticity

$$\ddot{\mu}^{\theta}_{ij} = -\xi^T \partial_{\theta_{ij}}\varepsilon$$

Synaptic gain

$$\ddot{\mu}^{\gamma}_{i} = \tfrac{1}{2} tr(R_i(\xi\xi^T - \Pi(\mu^{\gamma})))$$

Birdsong and Attractors

- The empirical measures we focus are local field potentials (LFPs) or evoked (ERP) responses that can be recorded noninvasively.
- Our aim is to show that canonical features can be reproduced easily under attractor models of sensory input.

Attractors in the Brain

- The basic idea here is that the environment unfolds as an ordered sequence of spatiotemporal dynamics, whose equations of motion entail attractor manifolds that contain sensory trajectories.
- a **manifold** is a topological space that on a small enough scale resembles the Euclidean space of a specific dimension, called the dimension of the manifold. Thus, a line and a circle are one-dimensional manifolds, a plane and sphere (the surface of a ball) are two-dimensional manifolds, and so on into high-dimensional space.

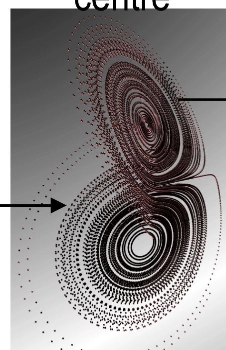
Synthetic song-birds

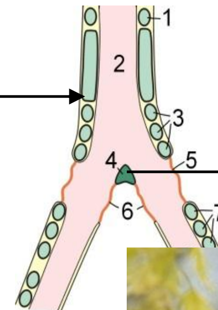
Vocal centre

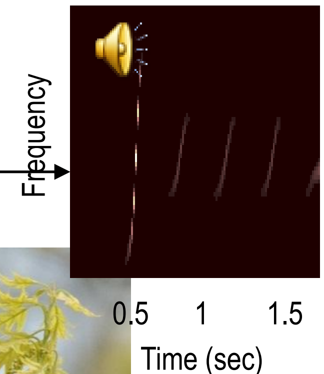
Syrinx

Sonogram

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

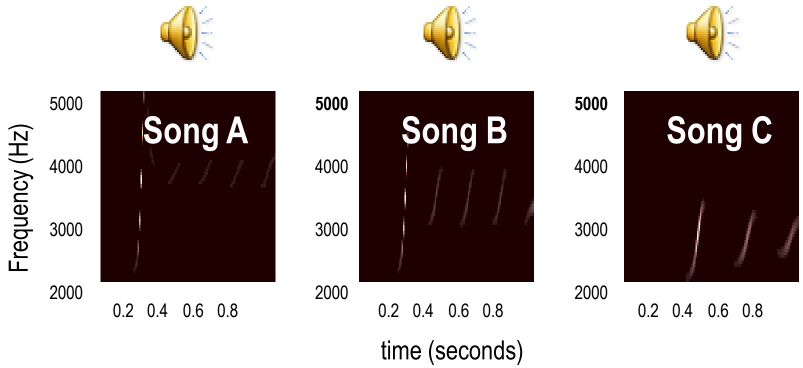


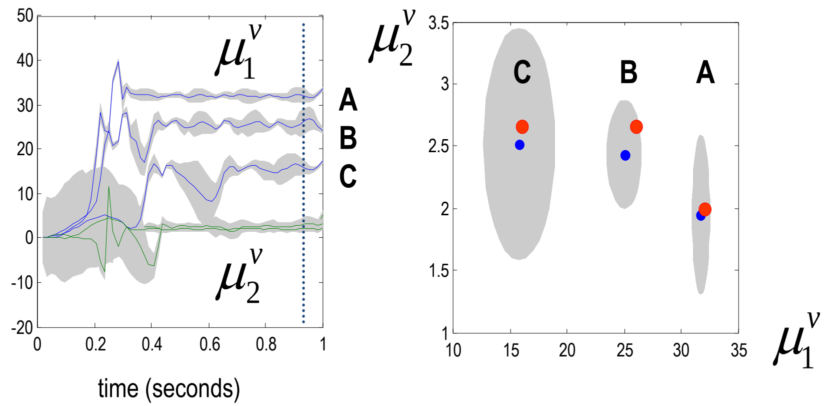




$$\dot{x} = \mathbf{f}(x,v) = \begin{bmatrix} 18x_2 - 18x_1 \\ v_1 x_1 - 2x_3 x_1 - x_2 \\ 2x_1 x_2 - v_2 x_3 \end{bmatrix}$$

Perceptual categorization





Attractors in the Brain (1/2)

- First, at any level the model can generate and therefore encode structured sequences of events, as the states flow over different parts of the manifold.
- Second, hierarchical deployed attractors enable the brain to generate and therefore predict or represent different categories of sequences.

Attractors in the Brain (2/2)

- Third, if the state of a higher attractor changes the manifold of a subordinate attractor, then the states of the higher attractor come to encode the category of the sequence or dynamics represented by the lower attractor.
- Finally, this particular model has implications for the temporal structure of perception. Put simply, the dynamics of high-level representations unfold more slowly than the dynamics of lower level representations.