Predictive Coding: A Free-Energy Formulation

KJ Friston and S Kiebel

11.11.15.(Tue)
Summarized by Joon Shik Kim
Quiz

• What is the most important characteristic of the cortical system in the brain?. Please give an answer with the consideration of the structure of multilayered neural network and 6 layers in cortex.
Introduction

• Brain uses empirical Bayes for inference about its sensory input, given the hierarchical organization of cortical systems.
Hierarchical Dynamic Models (1/4)

- Hierarchical dynamic models are probabilistic generative models
  \[ p(y, \mathcal{G}) = p(y | \mathcal{G}) p(\mathcal{G}), \]
  where \( y \) is the sensory data and \( \mathcal{G} = \{x, \nu, \Theta\} \)
  are parameters.
- \( \nu \): inputs, sources, causes, or hidden states.
- \( x \): mediate the influence of the input on the output and endow the system with memory.
Hierarchical Dynamic Models (2/4)

\[ y = g(x, v) + z \]
\[ \dot{x} = f(x, v) + w \]
Hierarchical Dynamic Models (3/4)

- $\tilde{y} = [y, y', y'', \ldots]^T$: generalized response comprising the position, velocity, acceleration jerk, and so on.

\[
\begin{align*}
    y &= g(x, v) + z \\
    y' &= g_x x' + g_v v' + z' \\
    y'' &= g_x x'' + g_v v'' + z'' \\
    \tilde{y} &= \tilde{g} + \tilde{z}
\end{align*}
\]

\[
\begin{align*}
    \dot{x} &= x' = f(x, v) + w \\
    \dot{x}' &= x'' = f_x x' + f_v v' + w' \\
    \dot{x}'' &= x''' = f_x x'' + f_v v'' + w'' \\
    D\tilde{x} &= \tilde{f} + \tilde{w}
\end{align*}
\]
Hierarchical Dynamic Models (4/4)

\[ p(\tilde{y}, \tilde{x}, \tilde{v}) = p(\tilde{y} | \tilde{x}, \tilde{v}) p(\tilde{x}, \tilde{v}) \]

\[ p(\tilde{y} | \tilde{x}, \tilde{v}) = N(\tilde{y} : \tilde{g}, \Sigma^v) \]

\[ p(\tilde{x}, \tilde{v}) = p(\tilde{x} | \tilde{v}) p(\tilde{v}) \]

\[ p(\tilde{x} | \tilde{v}) = N(D\tilde{x} : \tilde{f}, \Sigma^x) \]

- Plausible assumptions about noise are exploited by the brain.
- \( \tilde{\Sigma}^v, \tilde{\Sigma}^x \): covariances
- \( \tilde{\Pi}^v, \tilde{\Pi}^x \): precisions which are the inverse of the covariances.
Model Inversion and Variational Bayes (1/5)

• Variational Bayes, which is a generic approach to model inversion that approximates the conditional density $p(\mathcal{G} | \tilde{y}, m)$ on some model parameters, $\mathcal{G}$, given model $m$ and data $\tilde{y}$.

• The log-evidence can be expressed in terms of a free-energy and divergence term

\[
\ln p(\tilde{y} | m) = F + K(q(\mathcal{G}) \| p(\mathcal{G} | \tilde{y}, m)) \Rightarrow F = \langle \ln p(\tilde{y}, \mathcal{G}) \rangle_q - \langle \ln q(\mathcal{G}) \rangle_q
\]
Model Inversion and Variational Bayes (2/5)

• The objective is to optimize the sufficient statistics of $q(\mathcal{G})$ by maximising the free-energy and minimizing the divergence.

• In physics, the free-energy is to be minimized and the free-energy definition in this paper is the negative of the conventional free-energy.
Model Inversion and Variational Bayes (3/5)

- We now seek $q(u)$ that maximizes the free energy at each point of time. This is proportional to $\exp(V)$, where $V(t)$ is called the variational energy; this is the Gibbs energy expected under the conditional density of the parameters.
- $V(t) = U(t) = \ln p(\tilde{y}, u)$, where $u(t) = [\tilde{x}, \tilde{v}]^T$.
- Under the Laplace approximation, the conditional density assumes a fixed Gaussian form $q(u) = N(u : \tilde{\mu}, C)$. 
The conditional precision is a function of the mean. Therefore, we can reduce model inversion to optimizing one sufficient statistics; namely, the conditional mean. This is the solution to
\[ \dot{\mu} - D\mu = \partial_u V. \]

\( \dot{\mu} - D\mu \) can be regarded as motion in a frame of reference that moves along the trajectory encoded in generalized coordinates.
Model Inversion and Variational Bayes (5/5)

- If the gradient of the variational energy is zero, the mean of the motion becomes the motion of the mean, $\dot{\mu} = D\mu$.

frame of reference that moves along the trajectory
Hierarchical Models in the Brain

• A key architectural principle of the brain is the hierarchical organization.
• Forward connections arise largely in superficial pyramidal cells, in supra-granular layers, and terminate on spiny stellate cells of layer four in higher cortical areas.
• Backward connections arise largely from deep pyramidal cells in infra-granular layers and target cells in the infra and supra-granular layers of lower cortical areas.
Perception and message-passing:
\[
\xi^{(i)v} = \Pi^{(i)v} \epsilon^{(i)v} = \Pi^{(i)v} (\mu^{(i-1)v} - g(\mu^{(i)})) \\
\xi^{(i)x} = \Pi^{(i)x} \epsilon^{(i)x} = \Pi^{(i)x} (D \mu^{(i)x} - f(\mu^{(i)}))
\]

Forward prediction error:

Synaptic plasticity:
\[
\ddot{\mu}_{ij}^{\theta} = -\xi^T \nabla \theta \epsilon
\]

Backward predictions:

Synaptic gain:
\[
\ddot{\mu}_i^{\gamma} = \frac{1}{2} tr(R_i (\xi \epsilon^T - \Pi(\mu^\gamma)))
\]
Birdsong and Attractors

• The empirical measures we focus are local field potentials (LFPs) or evoked (ERP) responses that can be recorded noninvasively.

• Our aim is to show that canonical features can be reproduced easily under attractor models of sensory input.
Attractors in the Brain

• The basic idea here is that the environment unfolds as an ordered sequence of spatiotemporal dynamics, whose equations of motion entail attractor manifolds that contain sensory trajectories.

• a manifold is a topological space that on a small enough scale resembles the Euclidean space of a specific dimension, called the dimension of the manifold. Thus, a line and a circle are one-dimensional manifolds, a plane and sphere (the surface of a ball) are two-dimensional manifolds, and so on into high-dimensional space.
Synthetic song-birds

\[ \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \]

\[ \dot{x} = \mathbf{f}(x, \mathbf{v}) = \begin{bmatrix} 18x_2 - 18x_1 \\ v_1x_1 - 2x_3x_1 - x_2 \\ 2x_1x_2 - v_2x_3 \end{bmatrix} \]
Perceptual categorization

![Graphs showing frequency vs. time for songs A, B, and C.](image-url)
Attractors in the Brain (1/2)

• First, at any level the model can generate and therefore encode structured sequences of events, as the states flow over different parts of the manifold.

• Second, hierarchical deployed attractors enable the brain to generate and therefore predict or represent different categories of sequences.
Attractors in the Brain (2/2)

• Third, if the state of a higher attractor changes the manifold of a subordinate attractor, then the states of the higher attractor come to encode the category of the sequence or dynamics represented by the lower attractor.

• Finally, this particular model has implications for the temporal structure of perception. Put simply, the dynamics of high-level representations unfold more slowly than the dynamics of lower level representations.