Ch2. Causal Inference in Sensorimotor Learning and Control

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Multisensory Predictive Learning

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Questions

- Why is causal inference important to cue combination and motor adaption?
- Why are mixture models more proper than linear models for causal inference?
Cue Combination and Causal Inference

- **Cue combination**
  - Sensory cues have noise.
  - Combining multiple cues reduce the effect of noise.

- **Causal Inference**
  - Sensory cues may be ambiguous to the nervous system.
  - Infer the causes of the cues and their relative contributions
    - Same causes or different ones
    - Process together or separately
  - How to infer properties of the body or world for perception and sensorimotor control under noisy cues
  - Bayesian method of causal-inference problems
Linear Models

- **Optimal cue combination with a common cause**
  - Linear combination of cues
    - Weight: the reliability of the individual cues ($1/\sigma^2$)
    - Assumption
      - Conditional independency between individual cues
      - Gaussian noise: cues are noisy copies of the source
      - Common cause: all cues are from the same source
  - Example: Object location problem
    - Infer location of the object with visual and auditory cues
    - Probability Distributions
      - Object location: $p(s) = N(\mu, \sigma_s)$
      - Visual cue ($v$) and auditory cue ($a$): $p(v \mid s) = N(s, \sigma_v)$, $p(a \mid s) = N(s, \sigma_A)$
      - Estimate $s$:
        $$p(s \mid v, a) = \frac{p(v, a \mid s)p(s)}{p(s)} = \frac{p(v \mid s)p(a \mid s)p(s)}{p(s)}$$
        - MAP estimation: $\hat{s} = w_v v + w_a a + w_\mu \mu$
        - Final estimates are more precise than the individual cues
Linear Models

Figure 2.1 Graphic representation of a causal-inference model. (Left) The visual and auditory cues can be from the same source $s$. This is the default scenario that the linear model of cue combination considers. (Right) Alternatively, the visual and auditory cues are from different sources. The mixture model of causal inference considers both scenarios and infers the probability of each case. The final estimate is a weighted average of the estimates from both cases with weights equal to their corresponding probabilities (Eq. 2.12).
Linear Models

 Violations of the linearity of cue combination

- Cues from different sources
- Spatiotemporal difference
  - The key factor to determine strength of cue integration
  - The loudspeaker far away from the screen or the stage in an outdoor movies or a concert
- Incorrect causal attribution
  - Leads to illusion
    - Three beeps vs. two flashes
    - Three beeps vs. touch twice
  - Spatiotemporal difference reduces the illusion
- Cues are not always combined linearly and incomplete integration is a function of spatiotemporal difference between cues.
Figure 2.2 Example of audiovisual cue combination. Subjects estimate the position of an auditory cue. The green distribution is the auditory likelihood. The red distribution is the visual likelihood. The blue distribution is the posterior distribution of the location of the auditory source. (A and B) When two cues are very similar to one another (in terms of spatial distance in this example), they will be combined in good approximation as though they have a common source as $p(c)$ is close to 1. (C) If there is more spatial disparity between the two cues, $p(c)$ can be close to 0.5. Under these circumstances both peaks are important and the mean-squared error function results in an intermediate estimate. (D) As the disparity increases further, $p(c)$ approaches 0 and the cues are processed independently.
Mixture Models for Casual Inference

❖ Optimal cue combination with unknown causal structure

● Determine whether to process cues together or separately → The nervous system should estimate the causal relationships between cues

● Experiment: Object location problem
  ▪ Each cue from different source to another
  ▪ Probability distribution
    • Visual cue (v) and auditory cue (a): \( p(v) = \mathcal{N}(s_v, \sigma_v), \ p(a) = \mathcal{N}(s_a, \sigma_a) \)
  ▪ Notation

<table>
<thead>
<tr>
<th>( s_v, s_a )</th>
<th>Actual location of visual and auditory stimuli</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v, a )</td>
<td>(Noisy) estimates obtained by the visual and auditory modalities separately</td>
</tr>
<tr>
<td>( \hat{s}_v, \hat{s}_a )</td>
<td>Optimal estimates of the positions of visual and auditory stimuli</td>
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</tbody>
</table>
Mixture Models for Casual Inference

- **Experiment**: Object location problem
  - **Common cause and uncommon cause probabilities**
    - Common: \( p(c \mid v, a) \)
    - Separate: \( p(\neg c \mid v, a) = 1 - p(c \mid v, a) \)
    - With Baye’s rule:
      \[
      p(c \mid v, a) = \frac{p(v, a \mid c) p(c)}{p(v, a)}, \quad p(\neg c \mid v, a) = \frac{p(v, a \mid \neg c) p(\neg c)}{p(v, a)}
      \]
  - **The joint probability**: \( p(v, a) = p(c) p(v, a \mid c) + p(\neg c) p(v, a \mid \neg c) \)
    - Not choose one among two cases but estimate the probability of each case to use the weight
    - Mixing components for a common cause and the uncommon causes
      \[
      p(v, a \mid c) = \int p(v, a \mid s) p(s) ds \quad p(v, a \mid \neg c) = \int p(v, a \mid s_v, s_a) p(s_v, s_a) ds_v ds_a
      \]
      \[
      = \int p(v \mid s) p(a \mid s) p(s) ds \quad = \int p(v \mid s_v) p(s_v) ds_v \times \int p(a \mid s_a) p(s_a) ds_a
      \]
    - **In case of Gaussian distribution**:
      \[
      p(v, a \mid c) = \frac{1}{2\pi \sqrt{\sigma_v^2 + \sigma_a^2 + \sigma_s^2}} \exp \left[ -\frac{1}{2} \left( \frac{(v-a)^2 \sigma_v^{-2} + (v-\mu)^2 \sigma_a^{-2} + (a-\mu)^2 \sigma_s^{-2}}{\sigma_v^2 + \sigma_a^2 \sigma_s^2} \right) \right]
      \]
      \[
      p(v, a \mid \neg c) = \frac{1}{2\pi \sqrt{(\sigma_v^2 + \sigma_s^2)(\sigma_a^2 + \sigma_s^2)}} \exp \left[ -\frac{1}{2} \left( \frac{(v-\mu)^2}{\sigma_v^2 + \sigma_s^2} + \frac{(a-\mu)^2}{\sigma_a^2 + \sigma_s^2} \right) \right]
      \]
Mixture Models for Casual Inference

● Example: Object location problem

  ▪ Optimal estimates

  \[ \hat{s}_c = \frac{v}{\sigma_v^2} + \frac{a}{\sigma_a^2} + \frac{\mu}{\sigma_s^2} \]

  \[ \hat{s}_{v,-c} = \frac{v}{\sigma_v^2} + \frac{\mu}{\sigma_s^2} \]

  \[ \hat{s}_{a,-c} = \frac{a}{\sigma_a^2} + \frac{\mu}{\sigma_s^2} \]

  ▪ Probability distribution of each source: mixture of two probabilities

  \[ p(s_A | v, a) = p(c | v, a)p(s_A | v, a, c) + p(\neg c | v, a)p(s_A | v, a, \neg c) \]

  \[ p(s_V | v, a) = p(c | v, a)p(s_V | v, a, c) + p(\neg c | v, a)p(s_V | v, a, \neg c) \]

  ▪ Best estimate of each cue

  \[ \hat{s}_v = p(c | v, a)\hat{s}_c + p(\neg c | v, a)\hat{s}_{-c,v} \]

  \[ \hat{s}_a = p(c | v, a)\hat{s}_c + p(\neg c | v, a)\hat{s}_{-c,a} \]

  ▪ Cue combination is nonlinear
Mixture Models for Casual Inference

❖ Causal inference in perception

**Figure 2.3** Experimental data (Wallace et al., 2004) and corresponding predictions from the mixture model of causal inference (Körding et al., 2007). (A) The relative frequency of subjects reporting one cause (black) is shown with the predictions of the causal-inference model (red). The probability of reporting a common cause depends on the spatial difference between the visual and auditory stimuli. (B) The bias, that is, the linear influence of vision on the perceived auditory position is shown (black). A bias of zero implies that vision has no influence. A bias of one would imply that subjects only use vision when estimating the auditory position. The predictions of the model are shown in red. Data from trials in which subjects report a common cause and trials in which subjects report independent causes are plotted separately. (C) A schematic illustration explaining the finding of negative biases. Blue and black dots represent the perceived visual and auditory stimuli, respectively. In the pink area people report having perceived a common cause.
Mixture Models for Casual Inference

Causal inference in motor learning and control

- Motor adaptation
  - How people respond to changes in their motor apparatus and in the environment
  - Motor system changes its behavior in response to movement errors
- Linear model
  - Movement change is linearly proportional to error (small error → small change, large error → large change)
  - Break down: it depends on the cause of error (extrinsic vs. intrinsic factors)
- Non-linear property
  - Small error → strong adaptation
  - Enough large error → weak adaptation
Mixture Models for Casual Inference

• Experiment: Straight reaching movement
  - Subjects are required to make straight reaching movements to a target in a virtual-reality setting where the visual feedback of the hand was represented as a cursor
  - Error (visual disturbance) = target – cursor
  - Cursor = movement + random perturbation
  - Cues: visual and proprioceptive cue
Mixture Models for Casual Inference

Experiment: Straight reaching movement

Figure 2.4 Experimental data and predictions from the mixture model of causal inference (Wei & Körding, 2008). Error bars denote standard errors over subjects. (A) Deviations of hand (from trials following perturbations; an indicator of adaptation) and the corresponding model predictions are plotted as a function of the size of visual disturbance (displacement of cursor relative to hand) for all subjects. Results from 5 and 15 cm movements are plotted separately. The deviations take the opposite sign as the disturbance, indicating adaptation aiming to compensate for errors. The adaptation is a nonlinear function of applied visual disturbance. The same disturbances elicit less adaptation when the hand movement is smaller (5 cm vs. 15 cm). (B) The normalized probability of visual error being relevant as a function of the size of the visual disturbance. This probability is equivalent to the conditional probability of a common cause given the sensory cues in the mixture model described earlier. The probability is highest when there is no visual disturbance (equivalent to the case of no difference between cues); it drops with increasingly larger disturbances. In the experiment with smaller movement amplitude (5 cm vs. 15 cm), the inferred probability of a common cause is smaller and it drops faster.
Mixture Models for Casual Inference

• Some other experiments
  ▪ Linear model cannot fully account for the nonlinearity
  ▪ Causal inference model
    • Less motor variance makes the nervous system infer the cause more easily
    • The adaptation is nonlinear function of error size

Figure 2.5 Empirical data from three motor-adaptation studies and the corresponding predictions from the mixture model of causal inference (Wei & Körding, 2008). Gray symbols are for data, and black symbols are for model predictions. (A) Study by Fine and Thoroughman (2007): The amount of adaptation in reaching movements is plotted as a function of the gain of the viscous perturbation (B) Study by Wei et al. (2005): The inverse of the adaptation rate in a visuomotor-adaptation task is plotted as a function of visual error gain. (C) Study by Robinson et al. (2003): The adaptation gain of saccades is plotted as a function of the visual error size.
Mixture Models for Casual Inference

- **Attribution of errors to multiple sources**
  - Real-world problems are more complex than bi-modal cues.
  - The nervous system should infer the complicated causes in the body and the environment that give rise to the observed error.
  - Traditional models
    - Do not assume learning of the nervous system from errors by estimating the sources
    - Use joint representation of the body and the environment
    - Generalization problem
  - Alternatives: source-estimation model
    - The model infer the causes of errors
    - The nervous system interprets movement errors in terms of the sources

- **Interaction-Prior Models**
  - An alternative model to mixture models
  - Do not use latent variables (underlying sources)
  - Estimates the distribution of the interaction prior via subjects’ response or makes assumptions about its specific form