The Dynamical Systems Model of the Simple Genetic Algorithm

5 Finite Populations and the Fixed-Point

The set of possible finite populations of size r forms a discrete lattice within the simplex Λ.

- the fixed-point population might not actually exist as a possible population
- where a finite population GA will end up?
- the population will randomly move to any other possible population with non-zero mutation, then can the GA converge?

Given a population p, operator G denotes:

$$G(p) = U \circ \mathcal{F}p = \frac{1}{\overline{f}(p)} USp.$$
$$p(t+1) = G(p(t)).$$

This equation describes three things:

1. The probability that each individual will be in the next population.
2. The expected next population.
3. The actual next population as $r \to \infty$.

The probability that the next population is q given that the current one is p is given by

$$r! \prod_{j=0}^{s-1} \frac{(G(p)_j)^{rq_j}}{(rq_j)!}.$$

The operator G is *continuous, so G will have a very similar effect on points that are close to each other in the simplex. for our discussion, use a simpler norm:*

$$||x|| = \sum_{i=0}^{s-1} |x_i|.$$

The force of G at point p is denoted by $||G(p) - p||$. Then, the GA is likely to spend some time at populations p such that $||p - p^*||$ is small, where p^* is the fixed-point of G. Because the force of G will be small for populations that are near to the fixed-point.

6 Metastable States

The GA will stay in the vicinity of the infinite population fixed-point. The same argument would apply to any part of the simplex where the force of G is small, even if these areas are nowhere near the fixed-point. Such areas are called metastable states. Every eigenvector v ($G(v) = v$) is a fixed-point of G. Thus, we simply calculate all the eigenvectors of US and scale them so that their components sum to 1. If any of these are close to the simplex then, by continuity of G, we would expect a metastable region in that part of the simplex nearest the fixed-point.