Modelling Genetic Algorithm Dynamics

1. Why Model?
 -gain insights into how things work.
2. What to Model
 -Simple GA applied to one-counting problem
3.How to model
3.1 Naming the parts
 encoding:$(S_1, S_2, \ldots S_L)$

 fitness:$E = \sum_{i=1}^{L} S_i$

 mutation: $S_i^{\alpha} \rightarrow S_i^{\alpha'} = \begin{cases} S_i^{\alpha} & probability u \\ 1 - S_i^{\alpha} & probability 1 - u \end{cases}$

 recombination: $S_i^{u} = \begin{cases} S_i^{\alpha} & probability a \\ S_i^{\beta} & probability 1 - a \end{cases}$

3.2 Modelling philosophy
 -uses only macroscopic variables:mean fitness, variance, skewness and correlation
 -assumes all other higher order statistics are zero.
 -consider ensemble average
3.3 A Zeroth order Model
 assumption:fitness distribution is gaussian
 Selection:

$$\rho_s(E) = 2\rho(E)\int_{-\infty}^{E} \rho(E')dE'$$

$$\mu_s = \int_{-\infty}^{\infty} E\rho_s(E) = \mu + \frac{\sigma}{\sqrt{\pi}}$$

$$\sigma_s^2 = \int_{-\infty}^{\infty} (E-\mu)^2\rho_s(E) = (1 - \frac{1}{\pi})\sigma^2$$

 Mutation:

$$\sigma_m^2 = u(1-u)L + (1-2u)^2\sigma^2$$

 Together:

$$\mu(t+1) = uL + (1-2u)\mu(t) + \frac{(1-2u)}{\sqrt{\pi}}\sigma(t)$$

$$\sigma^2(t+1) = u(1-u)L + (1-2u)^2(1 - \frac{1}{\pi})\sigma^2(t)$$

-final σ value:$\sigma^{2_{eq}} = u(1-u)L/(1-b)$

-final μ value ignoring transient period:$\mu(\infty) = L/2 + (1-2u)\sigma_{eq}/(2u\sqrt{\pi})$