3.4 Crossover Dynamics

crossover variables: $C_i^{uv} = \begin{cases} 1 & \text{if } site\ i\ in\ u\ is\ replaced\ by\ site\ i\ in\ v \\ 0 & otherwise \end{cases}$

string crossover can be expressed as: $x^u \rightarrow x^u + (x^v - x^u) C^{uv}$

Consider uniform crossover with probability a:

$\kappa_1^{(c)} = \kappa_1$

$\psi^{(c)} = \psi.$

$\kappa_2^{(c)} = \kappa_2 - 2a(1-a)[\kappa_2 - \frac{1}{4}(\sum_i J_i^2 - L\psi)]$

$\kappa_3^{(c)} = \kappa_3 - 3a(1-a)(\kappa_3 - \kappa_3^{LE})$

$\kappa_4^{(c)} = \kappa_4 - 2a(1-a)(2 - a(1-a))(\kappa_4 - \kappa_4^{LE})$

where the superscript LE denotes the cumulants of the fixed-point crossover, a distribution where every site is drawn indepently from a binomial

$P_{LE}(x_i^\mu = 1) = \frac{1}{N}\sum_v x_i^v \equiv \tau_i$

as we don't know how the $\tau_i$ depends upon the sites, the principle of maximum entropy must be applied.

$P_{ME}(|\, x_i^\mu \,|) = \Pi i P_{ME}(\tau_i |\, J_i)$

$P_{ME}(\tau_i |\, J_i) = Z_i^{-1} e^{-\lambda_1 J_i \tau_i - \lambda_2 J_i^2 \tau_i^2}$

3.5 Mutation Dynamics

$q^{(m)} = (1-2\gamma)^2 q$

$\kappa_1^{(m)} = \kappa_1 + 2\gamma(\kappa_1^{random} - \kappa_1)$

$\kappa_2^{(m)} = \kappa_2 + 4\gamma(1-\gamma)(\kappa_2^{random} - \kappa_2)$

where the superscript random denotes cumulants of a random population.

3.6 Putting it together

-Selection increases the mean, reduces the variance, and introduces skewness and higher cumulants. These higher cumulants mean that the high fitness tail of the cost distribution is not very populated.

-Crossover restores higher cumulants to the linkage equibrium value.

-The ability for crossover to have this effect is meditated by the degree of correlation in the population.

4. Application to Harder problems: A Review of the Literature

-Problems that can be analysed as the simple problem in this paper

:Subset sum problem

-Problems that can be treated as a function of a weighted sum of bits.

:Learning in a perceptron with binary weights

:Transitions between barriers in a multi-modal landscape.

-The question of finding the optimal parameters of GA search
:We can do greedy search using equations of motion built before.